Geomatics Engineering Department

Second Year Geomatics

Geodesy 2 (GED209)

Lecture No: 3

Reference Systems in Geodesy

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Overview Of Previous Lecture

- Basic Concepts
- Intersection
- Resection
- Three-Point Resection Problem
- Numerical Exercise
- Situations To Use Each Method
- Summary

Overview Of Today's Lecture

- What is a Coordinate System?
- Where we made our Observations?
- Coordinate Systems in Geodesy
- Relation between Different Systems
- Summary
- Take Home Assignment

Expected Learning Outcomes

- Gain an understanding of the basic elements of a coordinate system, such as reference points, axes, and units.
- Explore various coordinate systems used in geodesy, such as the geodetic coordinate system (latitude and longitude).
- Understand the Earth-centered, Earth-fixed (ECEF) reference frame used in geodesy and the global reference systems.
- Learn about coordinate transformations and conversion techniques that allow for the conversion between different coordinate systems, such as transforming between geodetic coordinates and Cartesian coordinates.
- Understand how coordinate systems are used to precisely define positions on the Earth's surface or in space.

What is a Coordinate System?

DEFINITION

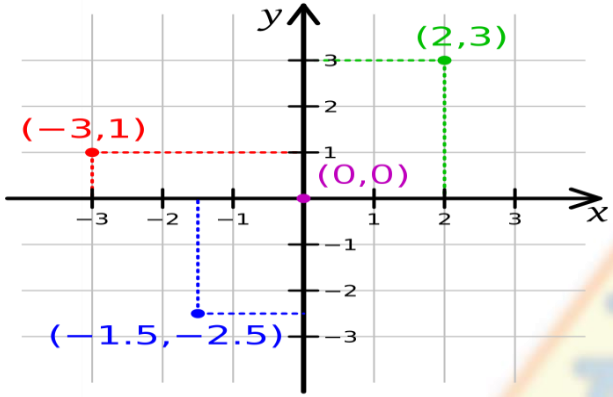
- ***A coordinate system*** is a mathematical framework used to determine the positions of points in space or on a surface.
- It provides a way to describe the location or movement of objects using numerical coordinates.
- **To define a coordinate system, we must specify:-**

a) the location of the origin,

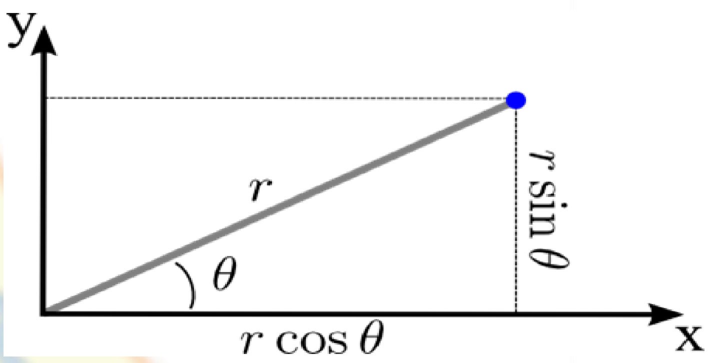
b) the orientation of the three axes,

c) the parameters (Cartesian, curvilinear) which define the position of a point referred to the coordinate system.

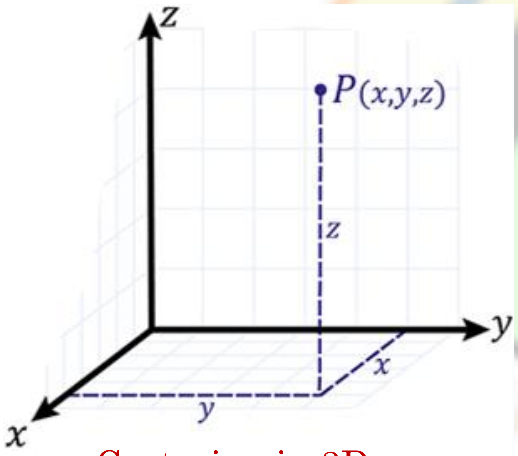
DEFINITION



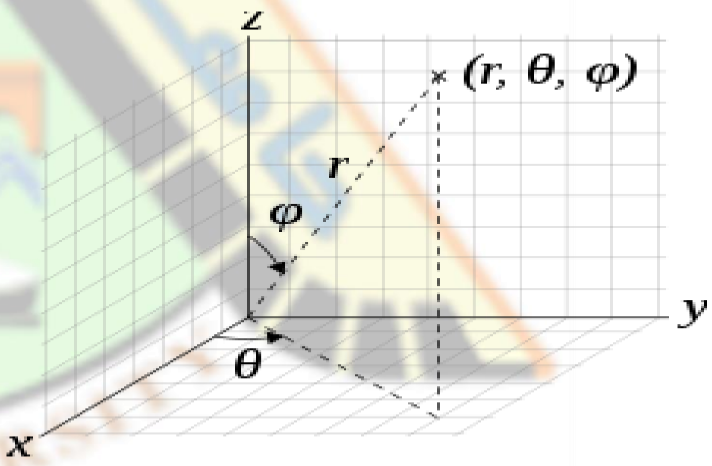
Cartesian in 2D



Polar in 2D



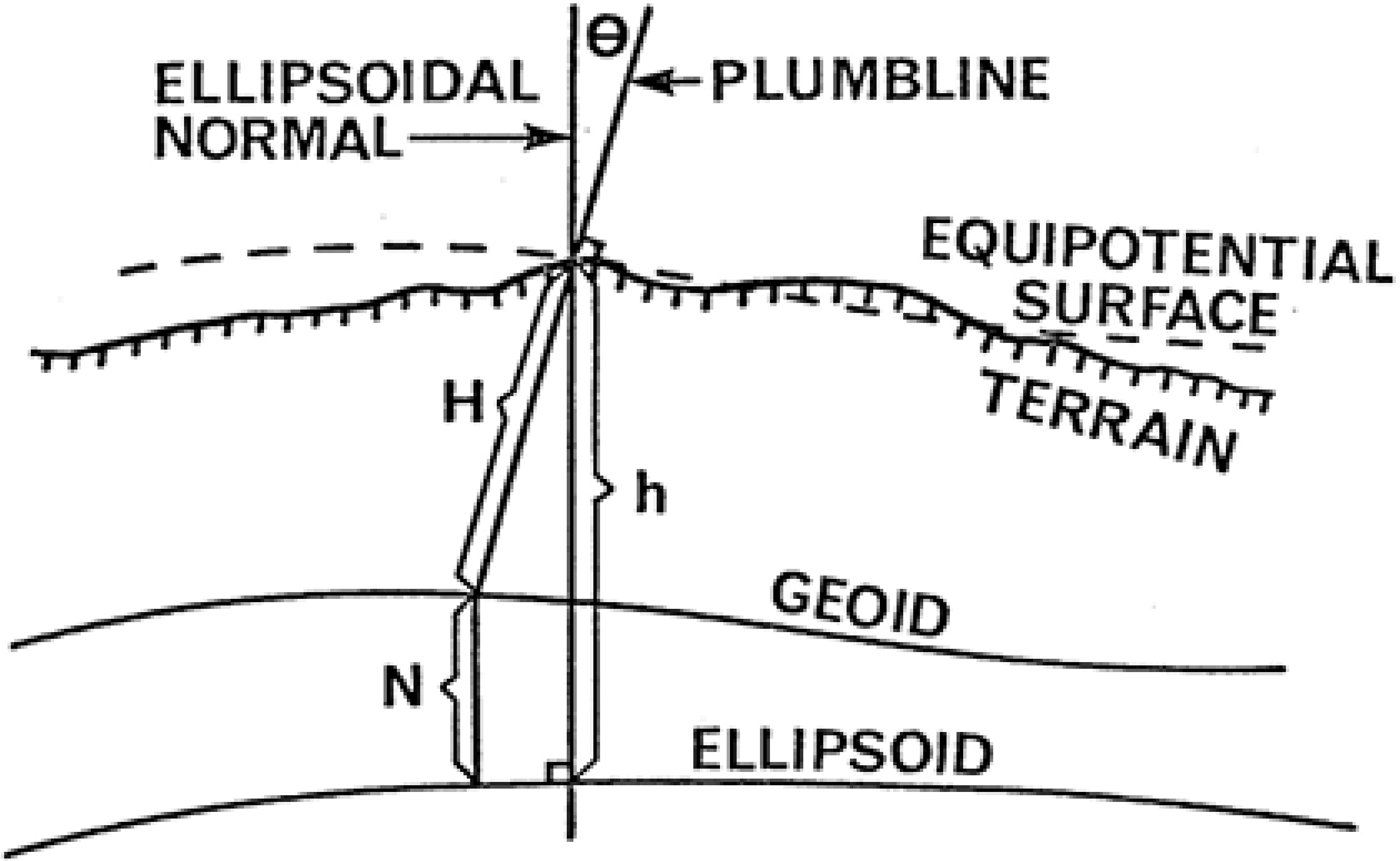
Cartesian in 3D



Polar in 3D

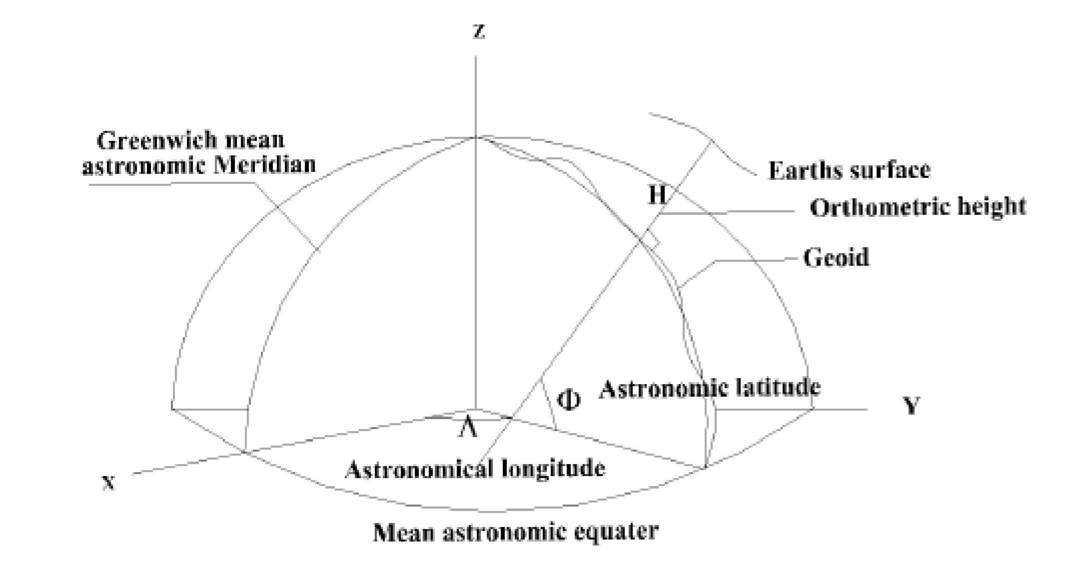
How Could This be Addressed in Geodesy?

WHERE WE MADE OUR OBSERVATIONS?



(1) Astronomic "Natural" Coordinate System

- Astronomical observations for latitude, longitude, and azimuths are measured with reference to the direction of gravity at the point of observation.
- In the natural coordinate system, the position of any point on the earth's surface can be fixed by observing its astronomic latitude Φ, longitude Λ, and its orthometric height H.

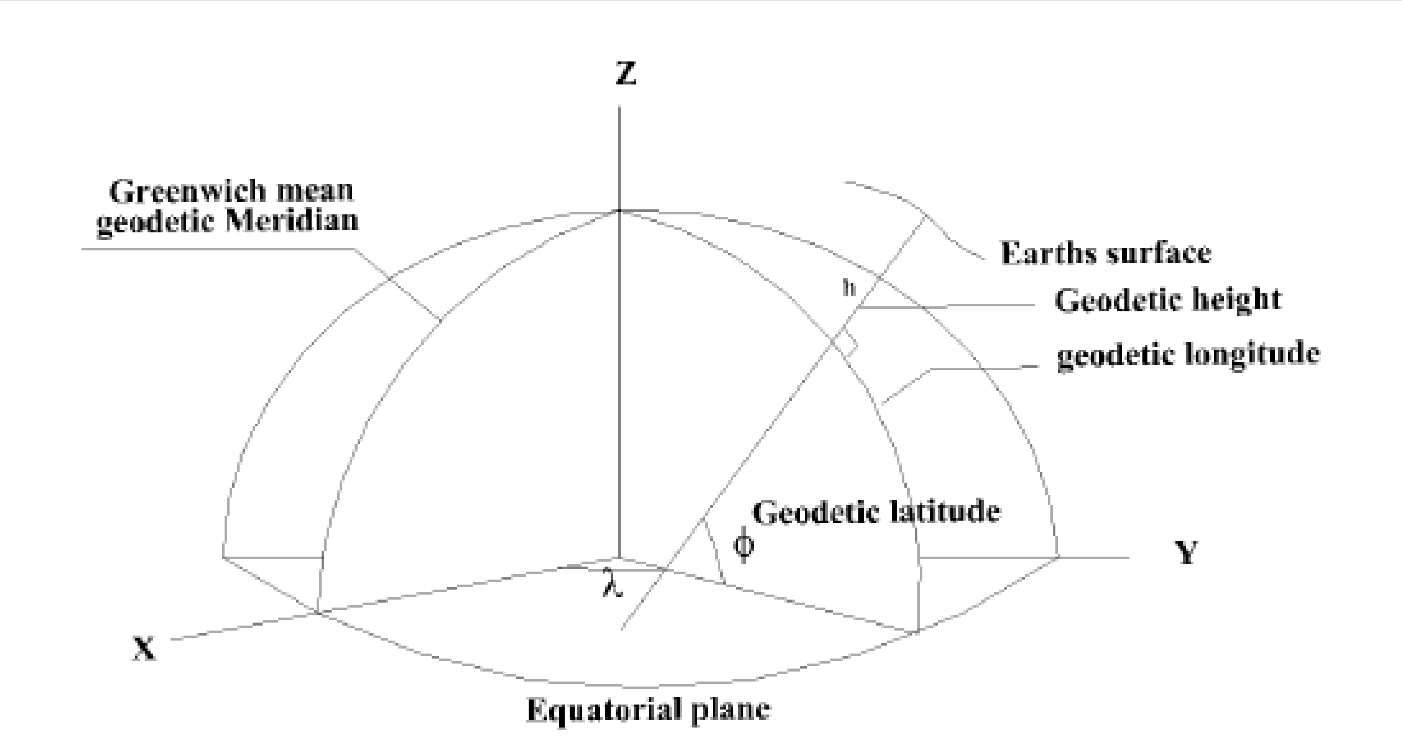


(1) Astronomic "Natural" Coordinate System

- **Astronomical Latitude** Φ: is the angle between the equatorial plane and the direction of the vertical at the point of observation.
- **Astronomical Longitude** Λ: is the angle between the meridian plane of the observation point and the meridian plane of Greenwich.
- **Orthometric Height** H: is the height of a point above mean sea level. It is measured along the curved plumb line and obtained from spirit levelling and gravity observations.
- The quantities Φ, Λ, and H define the position of the observer with respect to the geoid & the mean rotational axis of the earth.

(2) Geodetic Coordination System

- Since the deviations of the geoid from the reference ellipsoid are small and can be computed, it is convenient to add small reductions to the observed coordinate so that, values refer to an ellipsoid can be established, which are called ***geodetic coordinates***.

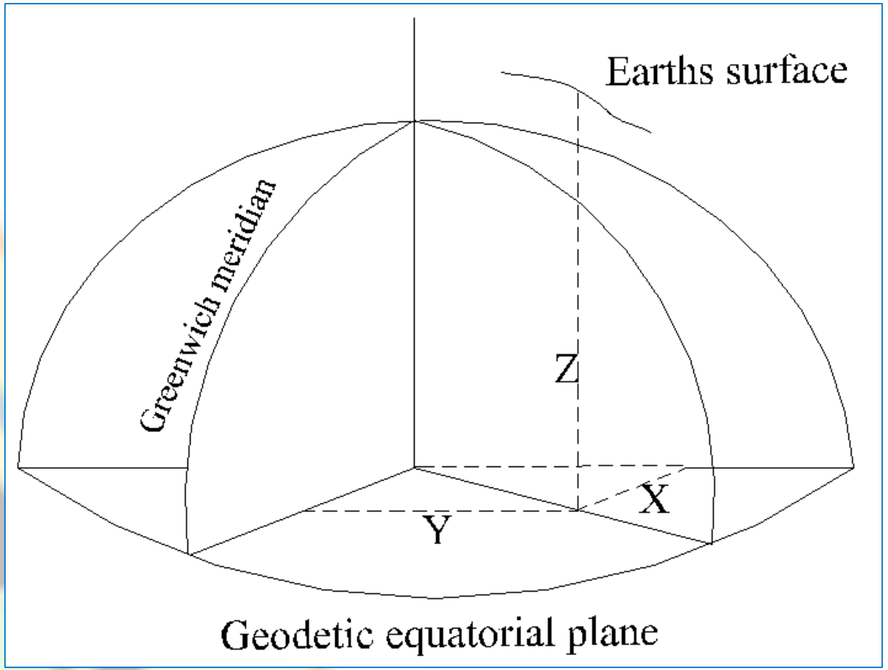


(2) Geodetic Coordination System

- **Geodetic Latitude** φ: is the angle between the ellipsoidal normal of the observers projected position on the geoid and the perpendicular to the mean rotation axis of the earth.
- **Geodetic Longitude** λ: is the angle between the same ellipsoidal normal and Greenwich meridian plane.
- **Geodetic Height** h: is the height of the observer above the reference ellipsoid, measured along the ellipsoidal normal.
- The geodetic coordinates are determined from Triangulation or Trilateration observed on the earth surface, reduced to the ellipsoid. They could also be obtained directly from the astronomic coordinates reduced to the used reference ellipsoid.

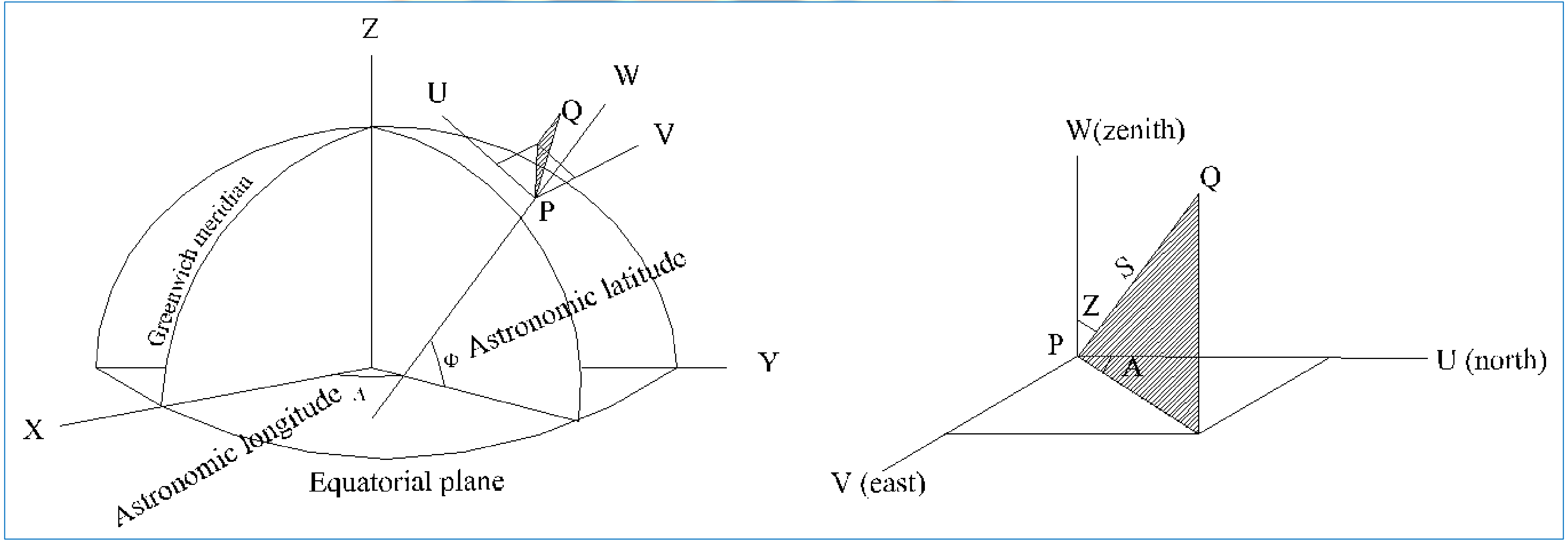
(3) Rectangular Coordinate System

- It is convenient to take the X-axis parallel to the meridian of Greenwich; the Y-axis has the longitude of 90° east of Greenwich, and the Z-axis parallel to the *CIO* (conventional international origin of polar motion).
- Ideally the origin of the rectangular coordinates system should be at the earth's center of gravity; the system is known as "*Average Terrestrial Coordinate System*".
- When the origin is at the geometric center of the ellipsoid, and not in the (C.G.) of the earth, it is known as "*Geodetic Coordinate System*".



(4) Local "Horizon " Coordinate System

- In this system the coordinates *U, V, W* are expressed as functions of the observed azimuth *A*, zenith distances *Z* & spatial distance *S*.
- The origin is at the observation station P. The positive U-axis points N-ward, the positive V-axis points eastward and the positive W-axis coincides with the outward direction of the plumb line.



(4) Local "Horizon" Coordinate System

- The coordinate equations of an object Q sighted in this system may be written simply as: -

$U = S \sin Z \cos A$ ______________________ (1)

$V = S \sin Z \sin A$ ______________________ (2)

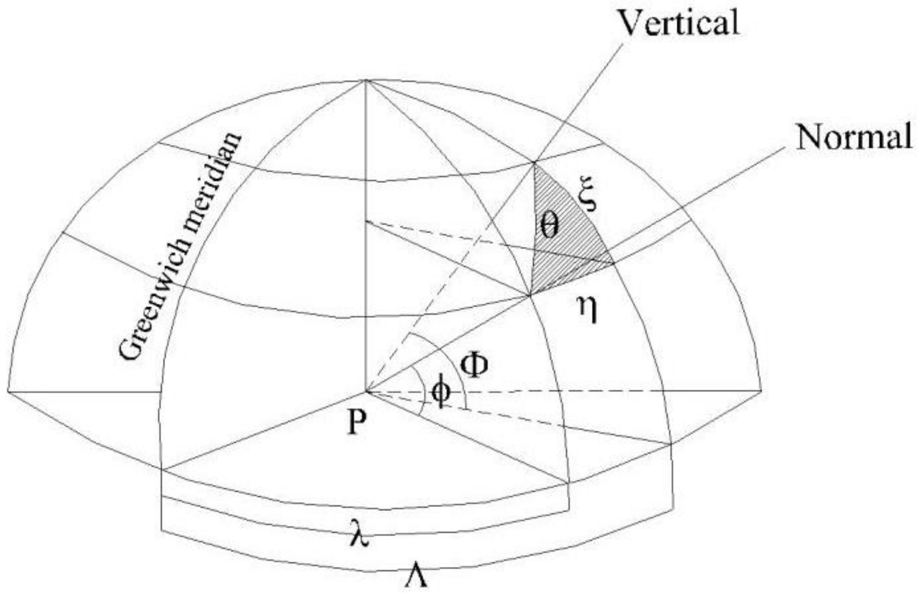
$W = S \cos Z$ ______________________ (3)

Are There Any Relations between different Reference Systems!

(1) Astronomic and Geodetic Systems

(1) Relation between Astronomic and Geodetic Coordinates

- The astronomical system depends on the direction of the vertical "*actual gravity field*", while the geodetic system depends on the direction of the ellipsoidal normal "*normal gravity field*", then the relation between both systems depends mainly on the difference between the two directions. The total difference between the two directions is the well-known *deflection of the vertical* θ. It has two components, a north-south component ξ and an east-west component η.



(1) Relation between Astronomic and Geodetic Coordinates

- The deflection components are given by: -

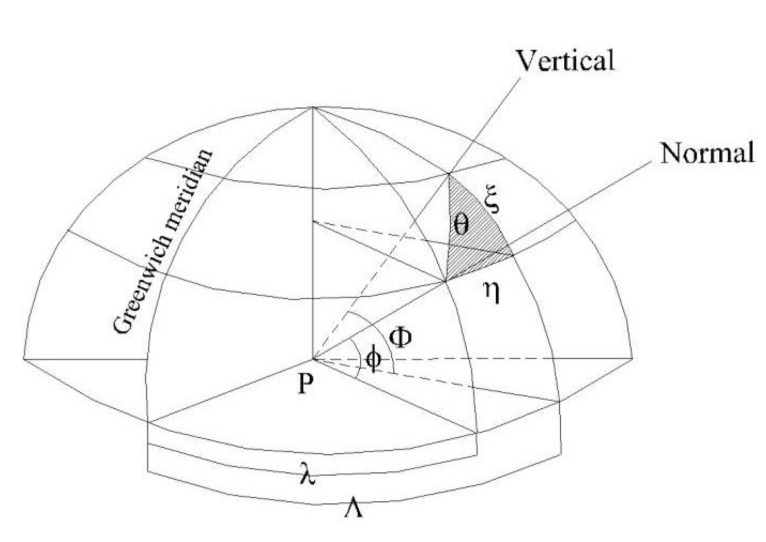
$$\xi = \Phi - \varphi \qquad (4)$$

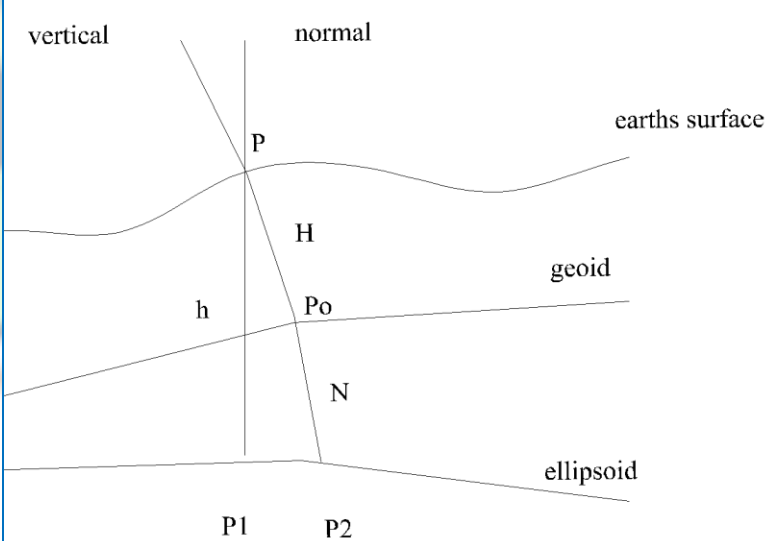
$$\eta = (\Lambda - \lambda)\cos\varphi \qquad (5)$$

$$\theta = \sqrt{\xi^2 + \eta^2} \qquad (6)$$

According to Helmert's projection, which neglects the curvature of the plumb line, a point P on the earth's surface is directly projected onto the ellipsoid by means of straight ellipsoidal normal, point P1. Then the ellipsoidal height is given by: -

$$h = H + N \qquad (7)$$



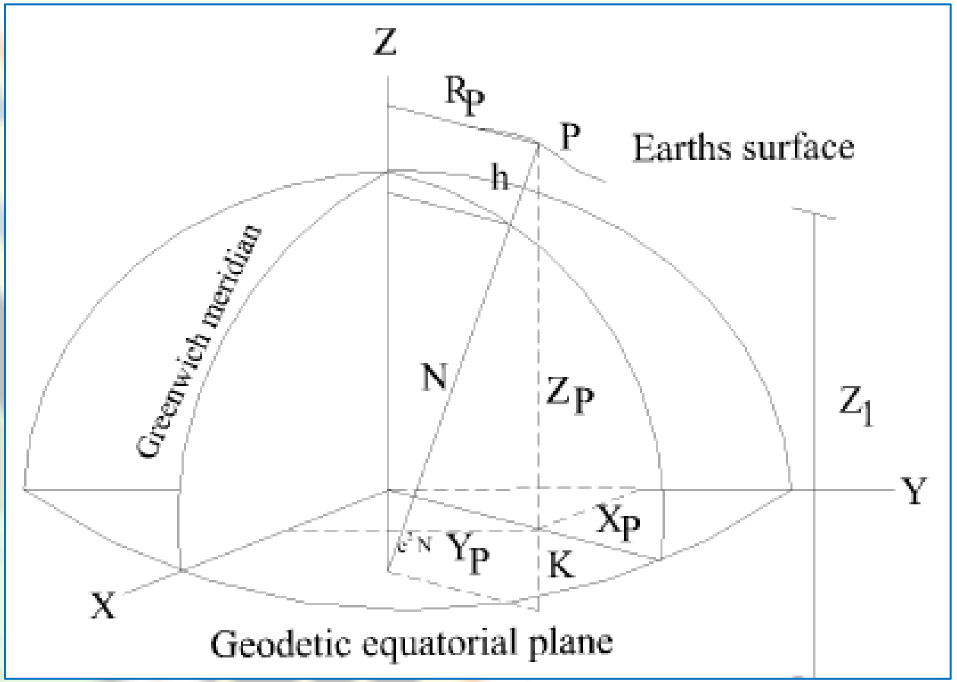


(2) Geodetic and Cartesian Systems

(2) Relation between Astronomic and Geodetic Coordinates

- The coordinate transformation between the curvilinear geodetic coordinates and the Cartesian coordinates may be expressed symbolically by: -

$$(\varphi, \lambda, h) \underset{(a,f)}{\longleftrightarrow} (X, Y, Z)$$



Direct Procedure

- From figure, the relation between the two systems can be written as follows: -

$$R_P = (N + h)\cos\varphi \quad (8)$$

$$X = R_P \cos\lambda \quad (9)$$

$$Y = R_P \sin\lambda \quad (10)$$

Such that N is the radius of curvature in the prime vertical, h is the ellipsoidal height.

$$N = \frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi)}}$$

(2) Relation between Astronomic and Geodetic Coordinates

- Also, from figure: -

$$z = Z_1 - K \qquad (11)$$

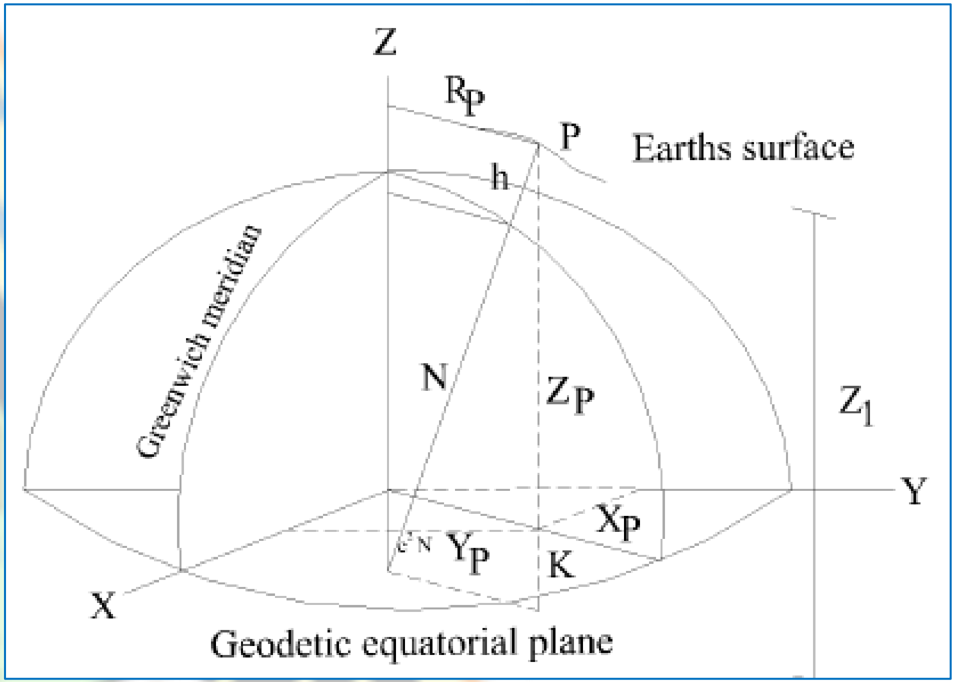
$$K = N \times e^2 \sin\varphi \qquad (12)$$

Combining the Eqs. 11, 12 with Eqs. 9 and 10.

$$X = (N + h)\cos\varphi\cos\lambda \qquad (13)$$

$$Y = (N + h)\cos\varphi\sin\lambda \qquad (14)$$

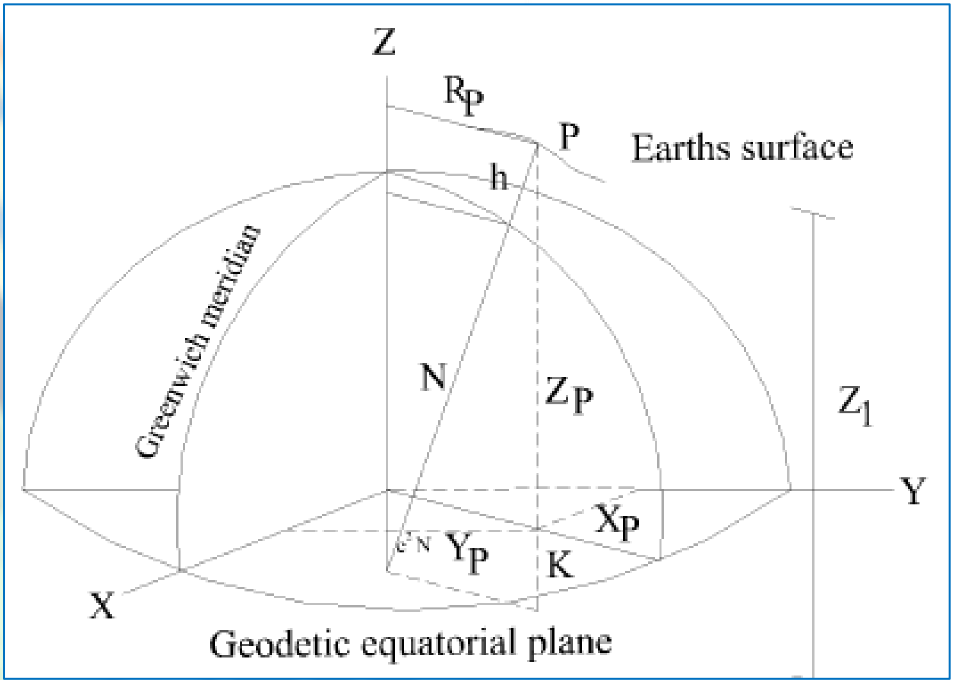
$$Z = (N(1 - e^2) + h)\sin\varphi \qquad (15)$$



(2) Relation between Astronomic and Geodetic Coordinates

- Inverse Procedure

The computation of φ, λ, h from given X, Y, Z is more complicated because the three equations have four unknowns, N including φ. Accordingly, the computation could be done iteratively in addition to the direct solution. Many solutions, through iteration, were given for this problem, for example; HIRVONEN & MORITZ 1963 BARTELME & MEISEL 1975, RAPP and KRAUSS 1976. Also, a non-iterative solution was given by SUENKEL 1976.



(2) Relation between Astronomic and Geodetic Coordinates

- Inverse Procedure

For the iterative solution we shall follow (Hirvonen & Moritz 1963).

From figure, we get: -

$$R_P = \sqrt{(X^2 + Y^2)} = (N + h)\cos\varphi, \quad \text{(16)}$$

therefore: -

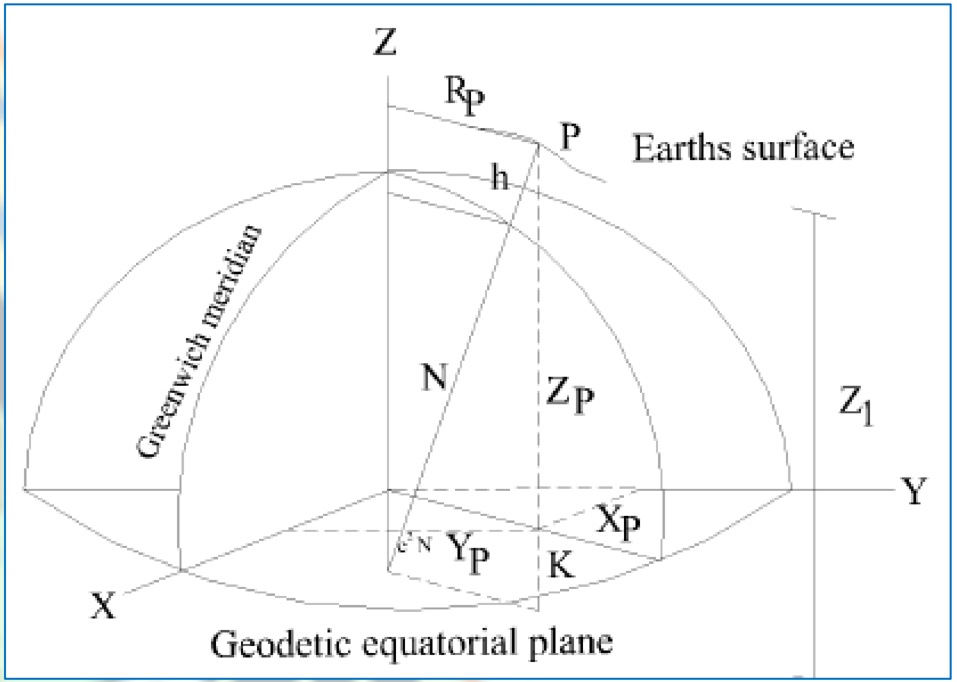
$$h = \frac{R_P}{\cos\varphi} N \quad \text{(17)}$$

Equation 15 may be transformed into: -

$$Z = \left(N - \frac{a^2 - b^2}{a^2} N + h\right)\sin\varphi$$

$$Z = (N + h - e^2 N)\sin\varphi, \quad \text{(18)}$$

such that $e^2 = \frac{a^2 - b^2}{a^2}$



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(2) Relation between Astronomic and Geodetic Coordinates

Inverse Procedure

Dividing equation 18 by equation 16, we get: -

$$\frac{Z}{R_P} = \left(1 - e^2 \frac{N}{N+h}\right) \tan\varphi, \text{ Then: -}$$

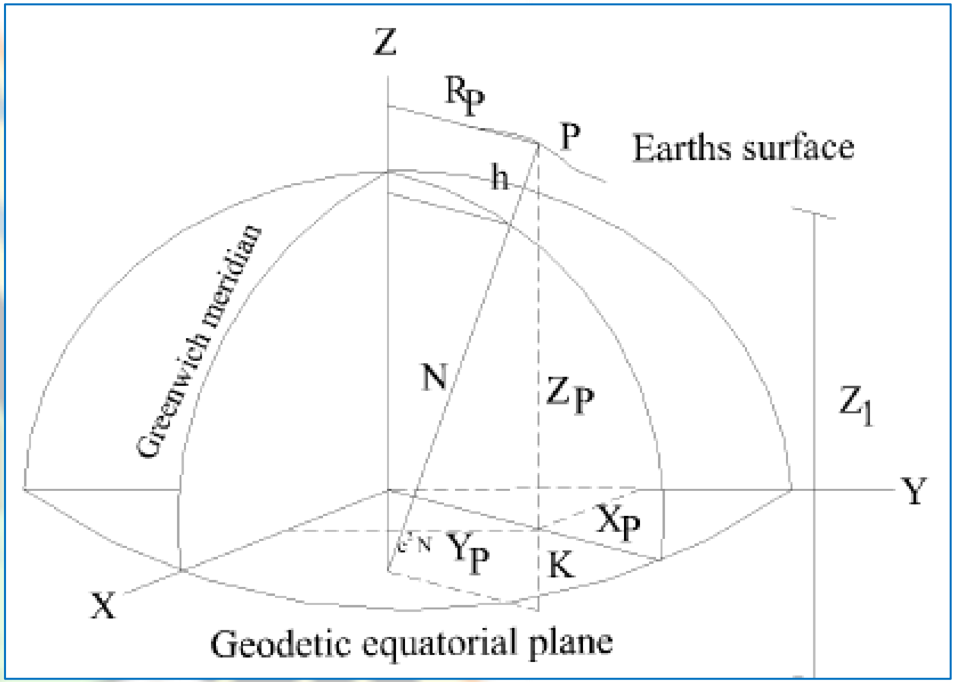
$$\tan\varphi = \frac{Z}{R_P}\left(1 - e^2 \frac{N}{N+h}\right)^{-1} \qquad (19)$$

Given X, Y, Z and hence, R_P equations 17 and 19 may be solved iteratively for h and φ.

As a first approximation, we set $h = 0$ in 19, obtaining: -

Using φ_1, we compute an approximate value N_1 by means of:-

$$N = \frac{a}{\sqrt{(1 - e^2 \sin^2\varphi_1)}} \qquad (20)$$



(2) Relation between Astronomic and Geodetic Coordinates

- Inverse Procedure

and introduce this value of N_1 in equation 17 to get an approximate value h_1.

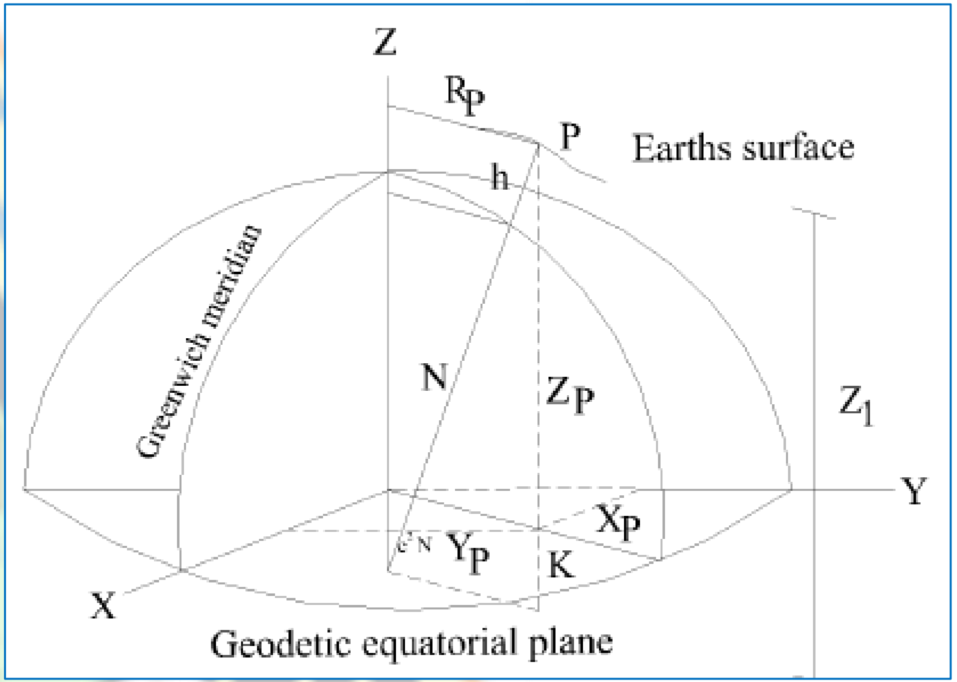
$$h_1 = \frac{R_P}{\cos\varphi_1} - N_1$$

Now, as a second approximation, we set $h_1 = h$ in 19 obtaining: -

$$\tan\varphi_2 = \frac{Z}{R_P}\left(1 - e^2\frac{N_1}{N_1+h_1}\right)^{-1}$$

Using φ_2, improved values for N & h are found, etc. This procedure is repeated until the values of φ & h remain practically constant. The third value λ can be easily calculated from: -

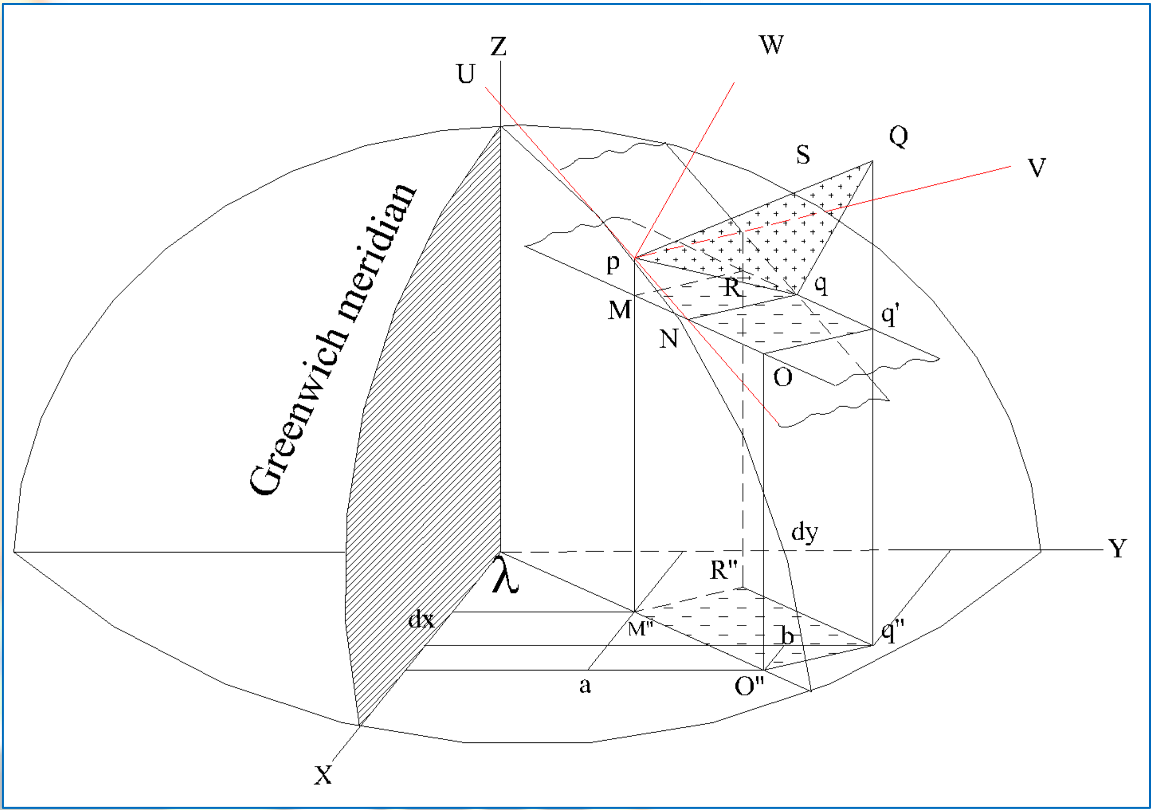
$$\tan\lambda = \frac{Y}{X} \qquad (21)$$



(3) Relation between Horizon and Rectangular System

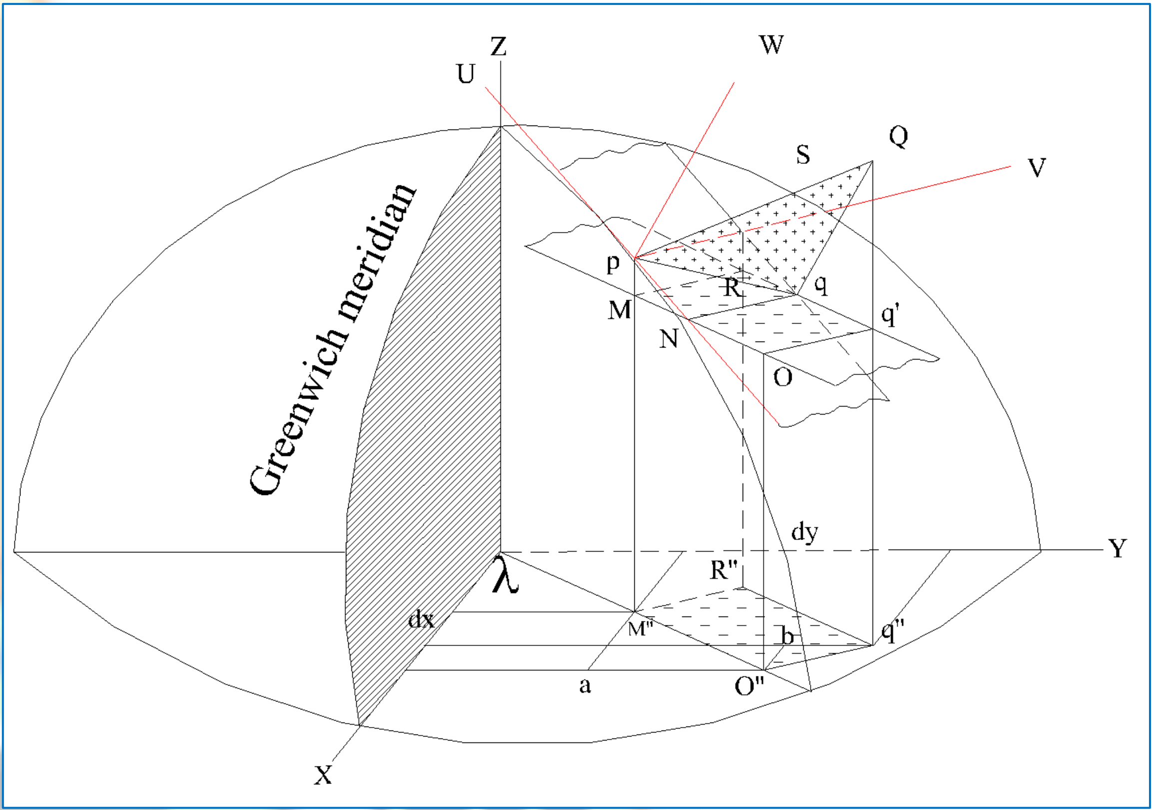
(3) Relation between Horizon and Rectangular System

- Since all the observations in geodesy, mainly horizontal, vertical angles, and spatial distances, are made with respect to the direction of the vertical at the observation station. Then it is important to find out the relations connecting these observable quantities of these two systems.
- The shown figure illustrates the quantities of these two systems, where point P represents the occupied station, Q is the observed objects, and pq is the horizontal projection of the spatial distance S onto the horizon plane ***Psqn*** of the local system U,V,W.



(3) Relation between Horizon and Rectangular System

- Using Equations 1, 2, and 3: $u, v,$ and w of station Q are computed w.r.t station P.
- The plane through points M, O, q' is parallel to the equatorial plane of the X, Y, Z system. The projection of this horizon plane on $MOq'R$ plane is given as follows: -



$$MO = NO + MN$$

$$MO = w\cos\Phi - u\sin\Phi \quad (22)$$

Likewise,

$$\Delta X = Ma - Ob = MO\cos\Lambda - v\sin\Lambda \quad (23)$$

$$\Delta Y = aO - bq = MO\sin\Lambda + v\cos\Lambda \quad (24)$$

$$\Delta X = -PM + qQ = u\cos\Phi + w\sin\Phi \quad (25)$$

(3) Relation between Horizon and Rectangular System

- Then, the final form are achieved by combining these relations together as follows: -

$$\begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} = \begin{vmatrix} -\sin\Phi\cos\Lambda & -\sin\Lambda & \cos\Phi\cos\Lambda \\ -\sin\Phi\sin\Lambda & \cos\Lambda & \cos\Phi\sin\Lambda \\ \cos\Phi & 0 & \sin\Phi \end{vmatrix} . \begin{vmatrix} u \\ v \\ w \end{vmatrix} \quad (26)$$

Or in matrix notation: -

$$X = R^T . u$$

Take Home Assignment

- Given the coordinates of 10 stations on WGS1984. Write a programming code to perform the coordinate conversion from one system to another.

Point ID	Latitude	Longitude	Ellipsoid Height	X (ECEF)	Y (ECEF)	Z (ECEF)	Geoid Height
GPS1	31.08918151	27.99223666	27.816	4827347.956	2565907.493	3274379.219	16.136
GPS2	31.08950619	27.98739942	28.105	4827548.372	2565491.325	3274410.195	16.150
GPS3	31.08349726	27.98509798	26.272	4827953.799	2565458.089	3273838.707	16.183
GPS4	31.08144478	27.98553466	40.702	4828048.905	2565555.813	3273651.267	16.190
GPS5	31.08092668	27.98773839	40.63	4827976.356	2565755.397	3273602.034	16.186
GPS6	31.08031383	27.99201563	44.408	4827818.635	2566133.79	3273545.791	16.175
GPS7	31.07479606	27.98735932	30.259	4828295.322	2565883.941	3273014.532	16.212
GPS8	31.07319155	27.9868811	32.061	4828399.181	2565887.454	3272863.095	16.220
GPS9	31.07515052	27.98446109	30.861	4828407.649	2565630.428	3273048.501	16.220
GPS10	31.06845986	27.98179744	43.791	4828874.796	2565590.783	3272419.805	16.256

Take Home Assignment

- Given the coordinates of 10 stations on WGS1984. Write a python code to perform the coordinate conversion from one system to another.
- **Each student is required to: -**

a) Write down his own code to perform coordinate conversion.

b) Generate a 2D scatter plot using the station coordinates.

c) Submit a detailed report including code, results, and analysis.

d) *A report should start with a title page indicating student Name, Number, Section No, etc.,*

e) **Deadline**: The reports should be delivered to your tutor on or before Tuesday 12th March 2024.

END OF PRESENTATION

THANK YOU FOR ATTENTION!